

# Global dynamo models from direct numerical simulations and their mean-field counterparts

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## ABSTRACT

**Context.** The recently developed test-field method permits to compute dynamo coefficients from global, direct numerical simulations. The subsequent use of these parameters in mean-field models enables us to compare self-consistent dynamo models with their mean-field counterparts. So far, this has been done for a simulation of rotating magnetoconvection and a simple benchmark dynamo, which are both (quasi-)stationary.

**Aims.** It is shown that chaotically time-dependent dynamos in a low Rossby number regime may be appropriately described by corresponding mean-field results. Also, it is pointed out under which conditions mean-field models do not match direct numerical simulations.

**Methods.** We solve the equations of magnetohydrodynamics (MHD) in a rotating, spherical shell in the Boussinesq approximation. Based on this, we compute mean-field coefficients for several models with the help of the previously developed test-field method. The parameterization of the mean electromotive force by these coefficients is tested against direct numerical simulations. In addition, we use the determined dynamo coefficients in mean-field models and compare the outcome with azimuthally averaged fields from direct numerical simulations.

**Results.** The azimuthally and time averaged electromotive force in fast rotating dynamos is sufficiently well parameterized by the set of determined mean-field coefficients. In comparison to the previously considered (quasi-)stationary dynamo, the chaotic time-dependence leads to an improved scale separation and thus to a better agreement between direct numerical simulations and mean-field results.

## 1. Introduction

Mean-field electrodynamics provides conceptual understanding of dynamo processes generating coherent, large-scale magnetic fields in planets, stars and galaxies (Krause & Rädler 1980; Moffatt 1978). But, it suffers from a number of free parameters, which are only poorly known under astrophysically relevant conditions. We briefly summarize the essentials of mean-field theory in order to specify this point. Within a mean-field approach, the velocity and the magnetic field are usually split in large-scale mean fields,  $\bar{\mathbf{v}}$ ,  $\bar{\mathbf{B}}$ , and residual parts,  $\mathbf{v}$ ,  $\mathbf{b}$ , varying on much shorter length or time scales,

$$\mathbf{V} = \bar{\mathbf{v}} + \mathbf{v}, \quad \mathbf{B} = \bar{\mathbf{B}} + \mathbf{b}. \quad (1)$$

Then, the evolution of the mean field is governed by

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times (\mathcal{E} + \bar{\mathbf{v}} \times \bar{\mathbf{B}} - \eta \nabla \times \bar{\mathbf{B}}). \quad (2)$$

In the above equation, the mean electromotive force is defined as  $\mathcal{E} = \overline{\mathbf{v} \times \mathbf{b}}$  and  $\eta$  is the magnetic diffusivity. Equation (2) is coupled with an equation for the residual field

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{G} + \bar{\mathbf{v}} \times \mathbf{b} + \mathbf{v} \times \bar{\mathbf{B}} - \eta \nabla \times \mathbf{b}), \quad (3)$$

in which  $\mathbf{G}$  is defined as  $\mathbf{G} = \mathbf{v} \times \mathbf{b} - \overline{\mathbf{v} \times \mathbf{b}}$ . The above system of equations is equivalent to the usual induction equation and no approximation has been applied so far. An important simplification of the mean-field approach consists in considering equation

(2) only, which requires to express  $\mathcal{E}$  as a linear functional of  $\mathbf{v}$ ,  $\bar{\mathbf{v}}$  and  $\bar{\mathbf{B}}$ . This is justified with regard to (3), and we may write

$$\mathcal{E}_i = \mathcal{E}_i^{(0)} + \int \int K_{ij}(\mathbf{x}, \mathbf{x}', t, t') \bar{\mathbf{B}}_j(\mathbf{x}', t') d^3 x' dt', \quad (4)$$

in which  $K$  denotes some integral kernel and we refer to cartesian coordinates for the moment. The term  $\mathcal{E}^{(0)}$  in (4) accounts for small-scale dynamo action and is not considered here; it will be discussed later. In addition, we assume that  $\mathcal{E}$  depends only instantaneously and nearly locally on  $\bar{\mathbf{B}}$ . This is a crucial assumption, it will be referred to as the assumption of scale separation in the following. Therefore,  $\bar{\mathbf{B}}$  in (4) may be replaced by its rapidly converging Taylor series expansion at  $\mathbf{x}$ ,

$$\bar{\mathbf{B}}_j(\mathbf{x}', t) = \bar{\mathbf{B}}_j(\mathbf{x}, t) + (x'_k - x_k) \frac{\partial \bar{\mathbf{B}}_j(\mathbf{x}, t)}{\partial x_k} + \dots, \quad (5)$$

and taken out of the integral:

$$\mathcal{E}_i = a_{ij} \bar{\mathbf{B}}_j + b_{ijk} \frac{\partial \bar{\mathbf{B}}_j}{\partial x_k} + \dots, \quad (6)$$

with

$$a_{ij} = \int \int K_{ij}(\mathbf{x}, \mathbf{x}', t, t') d^3 x' dt', \quad (7)$$

$$b_{ijk} = \int \int K_{ij}(\mathbf{x}, \mathbf{x}', t, t') (x'_k - x_k) d^3 x' dt' \quad (8)$$

...

The dynamo coefficients  $\mathbf{a}$  and  $\mathbf{b}$  defined in (7) and (8) together with the expansion (6) may finally be used to solve equation (2). Moreover,  $\mathbf{a}$  and  $\mathbf{b}$  are directly linked to dynamo processes, i.e. to the generation, advection and diffusion of the mean magnetic field, and thus provide physical concepts to explain dynamo action (e.g. Rädler 1995). Unfortunately, they are not known in general and previous work often relies on arbitrary assumptions on them.

The test-field method by Schinnerer et al. (2005, 2007) permits to compute  $\mathbf{a}$  and  $\mathbf{b}$  from direct numerical simulations. It was first applied to a simulation of rotating magnetoconvection (Olson et al. 1999) and a simple geodynamo simulation (Christensen et al. 2001), which are both stationary except for an azimuthal drift. In the past three years, the test-field method was intensively used to compute mean-field coefficients for box simulations (e.g. Sur et al. 2008; Gressel et al. 2008; Brandenburg 2009; Käpylä et al. 2009; Rädler & Brandenburg 2009), which are not in the focus of this paper. Although the test-field method proved to be reliable, the parameterization of the electromotive force for the steady geodynamo model was not satisfactory. The expansion in (6) does not converge for this steady example because of a non-sufficient scale separation (Schinnerer et al. 2007). In this paper, we revisit the problem for chaotically time-dependent dynamos. We test azimuthally and time averaged electromotive-force vectors against their parameterization based on corresponding dynamo coefficients and compare azimuthally and time-averaged fields from direct numerical simulations with mean-field results.

## 2. Dynamo simulations

We solve the equations of magnetohydrodynamics (MHD) in the Boussinesq approximation for a conducting fluid in a rotating spherical shell as given by Olson et al. (1999),

$$E \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} - \nabla^2 \mathbf{V} \right) + 2\mathbf{z} \times \mathbf{V} + \nabla P = Ra \frac{\mathbf{r}}{r_o} T + \frac{1}{Pm} (\nabla \times \mathbf{B}) \times \mathbf{B} \quad (9)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B} \quad (10)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T. \quad (11)$$

The coupled differential equations for the the velocity  $\mathbf{V}$ , the magnetic field  $\mathbf{B}$ , and the temperature  $T$  are governed by four parameters. These are the Ekman number  $E = \nu/\Omega L^2$ , the (modified) Rayleigh number  $Ra = \alpha_T g_0 \Delta T L / \nu \Omega$ , the Prandtl number  $Pr = \nu/\kappa$  and the magnetic Prandtl number  $Pm = \nu/\eta$ . In these expressions,  $\nu$  denotes the kinematic viscosity,  $\Omega$  the rotation rate,  $L$  the shell width,  $\alpha_T$  the thermal expansion coefficient,  $g_0$  is the gravitational acceleration at the outer boundary,  $\Delta T$  stands for the temperature difference between the spherical boundaries,  $\kappa$  is the thermal and  $\eta = 1/\mu\sigma$  the magnetic diffusivity with the magnetic permeability  $\mu$  and the electrical conductivity  $\sigma$ . Besides these input parameters, we introduce the magnetic Reynolds number  $Rm = V_{rms} L / \eta$ , and the local Rossby number  $Ro_l = V_{rms} / (\Omega L) \cdot (\bar{l}/\pi)$  as important output parameters. In the latter expressions,  $V_{rms}$  denotes the rms-velocity and  $\pi/\bar{l}$  is the mean half-wavelength of the flow derived from the kinetic energy spectrum (Christensen & Aubert 2006).

No-slip mechanical boundary conditions were chosen for all simulations presented here and the magnetic field continues as a

potential field outside the fluid shell. Convection is driven by an imposed temperature difference between the inner and the outer shell boundary.

## 3. Computation of dynamo coefficients

In order to determine the dynamo coefficients  $\mathbf{a}$  and  $\mathbf{b}$ , we apply the test-field method explained in full detail in Schinnerer et al. (2007). The principal idea of this approach is to measure the mean electromotive force generated by the interaction of the flow with an arbitrarily imposed test field,  $\bar{\mathbf{B}}_T$ . The imposed test field appears as an inhomogeneity in equation (3),

$$\frac{\partial \mathbf{b}}{\partial t} - \nabla \times (\mathbf{G} + \bar{\mathbf{V}} \times \mathbf{b} - \eta \nabla \times \mathbf{b}) = \nabla \times \mathbf{v} \times \bar{\mathbf{B}}_T, \quad (12)$$

which is solved simultaneously with (9-11). The electromotive force due to a given  $\bar{\mathbf{B}}_T$  may then be computed as  $\mathcal{E}_T = \overline{\mathbf{v} \times \mathbf{b}}$  with  $\mathbf{b}$  resulting from (12). Finally,  $\mathcal{E}_T$  is used to solve relation (6) for the dynamo coefficients. In order to close the resulting system of linear equations and to determine all components of  $\mathbf{a}$  and  $\mathbf{b}$ , the numerical experiment (12) must be repeated several times with different fields  $\bar{\mathbf{B}}_T$ .

So far, averaged quantities labelled by an overbar are axisymmetric fields whereas  $\mathbf{v}$  and  $\mathbf{b}$  in (12) are non-axisymmetric residuals. For a stochastically stationary but nevertheless chaotically time-dependent flow,  $\mathcal{E}$  and also the mean-field coefficients vary in time. Thus, we introduce an additional time averaging indicated by brackets,  $\langle \dots \rangle$ , and write

$$\mathcal{E} \approx \mathbf{a} \langle \bar{\mathbf{B}} \rangle + \mathbf{b} \langle \nabla \bar{\mathbf{B}} \rangle + \dots \quad (13)$$

instead of (6). In the above equation,  $\mathcal{E}$  has been time averaged,

$$\mathcal{E} = \langle \overline{\mathbf{v} \times \mathbf{b}} \rangle, \quad (14)$$

and also  $\mathbf{a}$  and  $\mathbf{b}$  denote time averaged tensors. Furthermore, we interpret averages in (2) as combined azimuthal and time averages. Both is formally not correct. However, the approximations applied can be justified, if

- i) temporal fluctuations of the axisymmetric velocity and magnetic field are comparatively small, and
- ii) time averages of the non-axisymmetric residuals of the velocity and the magnetic field are negligible.

The above assumptions imply that azimuthal and time averaging act in a similar way and reinforce each other. As a consequence, mean-field coefficients are derived in this context from time averages of vectors  $\mathcal{E}_T$  resulting from (12). We emphasise that assumptions i) and ii) are not justified a priori but have to be tested in numerical simulations.

## 4. Comparison of mean-field results with direct numerical simulations

Mean-field coefficients determined from direct numerical simulations may be used in mean-field models. Subsequently, azimuthally and time-averaged magnetic fields resulting from three-dimensional, self-consistent models may be compared with their mean-field counterparts. Mean-field calculations presented here are based on equation (2) written as an eigenvalue problem,

$$\sigma \bar{\mathbf{B}} = \nabla \times \mathbf{D} \bar{\mathbf{B}}, \quad (15)$$

in which the linear operator  $\mathbf{D}$  is defined as

$$\mathbf{D} \bar{\mathbf{B}} = \bar{\mathbf{V}} \times \bar{\mathbf{B}} + \mathbf{a} \bar{\mathbf{B}} + \mathbf{b} \nabla \bar{\mathbf{B}} - 1/Pm \nabla \times \bar{\mathbf{B}}, \quad (16)$$

**Table 1.** Overview of the models considered, ordered with respect to their local Rossby number.

Model	$E$	$Ra$	$Pm$	Mean l	$Ro_l$	$Rm$	$\Delta\mathcal{E}$	$\lambda/(\eta/L^2)$
1	$1 \times 10^{-3}$	100	5	5	0.013	39	4.6	4.18
2	$1 \times 10^{-4}$	334	3	11	0.014	123	1.6	-7.05
3	$1 \times 10^{-4}$	334	2	11	0.015	86	1.4	-3.87
4	$3 \times 10^{-4}$	195	3	9	0.019	67	1.6	2.22
5	$3 \times 10^{-4}$	243	2	9	0.024	56	1.2	0.66
6	$3 \times 10^{-4}$	285	2	9	0.026	61	1.3	0.05
7	$3 \times 10^{-4}$	375	1.5	11	0.042	60	1.1	0.78

and  $\mathbf{a}$  and  $\mathbf{b}$  are time-averaged dynamo coefficients as described above. The eigenvalues  $\sigma$  lead to a time evolution of each mode proportional to  $\exp(\sigma t)$ . For more details concerning the eigenvalue calculation, we refer to Schinnerer et al. (2010b).

In addition, the parameterization of the electromotive force by dynamo coefficients, i.e. relation (6), may be tested against the mean electromotive force in direct numerical simulations. Again by virtue of (13-14), we compare  $\mathcal{E}$  as given in (14) and derived directly from numerical models with

$$\mathcal{E}^{(1)} = \mathbf{a} \langle \bar{\mathbf{B}} \rangle + \mathbf{b} \langle \nabla \bar{\mathbf{B}} \rangle, \quad (17)$$

in which  $\langle \bar{\mathbf{B}} \rangle$  and  $\langle \nabla \bar{\mathbf{B}} \rangle$  are also obtained from direct numerical simulations. Furthermore, we follow Schinnerer et al. (2007) and introduce

$$\Delta\mathcal{E} = \frac{\int_V |\mathcal{E} - \mathcal{E}^{(1)}| dV}{\int_V |\mathcal{E}| dV} \quad (18)$$

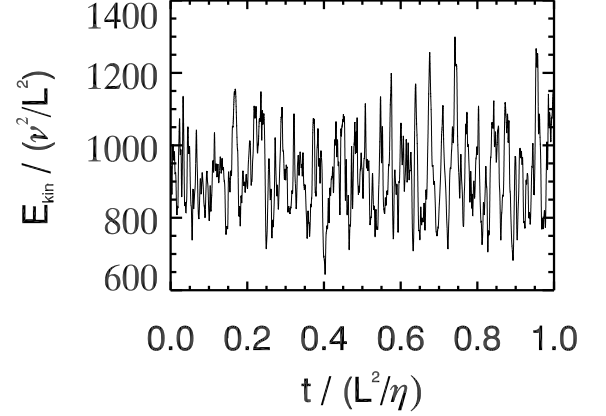
in order to quantify errors in the parameterization of  $\mathcal{E}$ .

## 5. Results

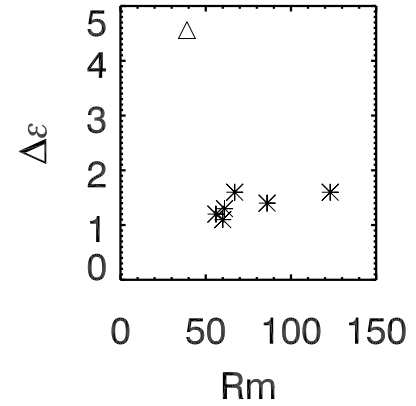
Besides the (quasi-)steady benchmark dynamo (model 1), we considered 6 chaotically time-dependent dynamos. The models are defined by 4 control parameters;  $E$ ,  $Ra$ , and  $Pm$  were varied (see Table 1), whereas the Prandtl number was kept fix and set to 1 for all simulations. The local Rossby number is always lower than 0.12 for the models considered here. They therefore belong to the regime of kinematically stable dynamos (Schinnerer et al. 2010a). In order to simplify the time averaging, models with low and fairly moderate  $Rm$  were chosen. The resulting velocity fields are almost symmetric with respect to the equatorial plane, except for model 7, and convection occurs mainly outside the inner core tangent cylinder. Figure 1 shows the volume-averaged kinetic energy density for model 3 and illustrates the irregular time-dependence of these models.

The parameterization of the mean electromotive force given by (17) improved a lot for the chaotically time-dependent dynamos in comparison to the benchmark dynamo;  $\Delta\mathcal{E}$  decreases by more than a factor of 4 from model 1 to model 7. The considerable drop of  $\Delta\mathcal{E}$  from the steady to the time-dependent models is most clearly visible in Fig. 2. It shows  $\Delta\mathcal{E}$  for different models versus their magnetic Reynolds number. Apart from the aforementioned difference between the time-dependent and the stationary models, a dependence of  $\Delta\mathcal{E}$  on  $Rm$  cannot be inferred.

In addition, the growth rate of the leading eigenmode of  $\mathbf{D}$ ,  $\lambda = \Re(\sigma)$ , may be taken as a measure for the accuracy of the mean-field description. Ideally, it is 0, whereas all overtones are highly diffusive (Schinnerer et al. 2010a, 2011b). For numerical simulations, however, it is impossible to hit the critical point exactly. The growth rates of the fundamental mode for all mod-



**Fig. 1.** Volume-averaged kinetic energy density versus magnetic diffusion time for model 3.

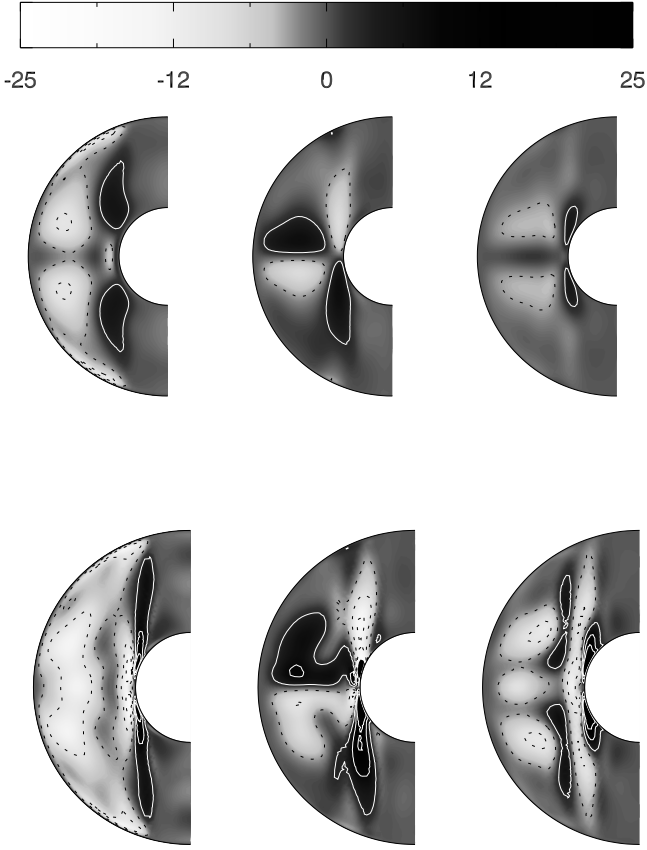


**Fig. 2.**  $\Delta\mathcal{E}$  versus the magnetic Reynolds number for the (quasi-)stationary benchmark dynamo (triangle) and the 6 chaotically time-dependent dynamos considered.

els are given in Table 1 in units of  $\eta/L^2$ . Note that the turbulent diffusivity largely exceeds the molecular one. For model 2, for instance, we find values of  $\beta$ -components up to  $63\eta$ . Thus,  $1/\lambda \approx 1/7 L^2/\eta$  for model 2 is in fact much larger than one effective decay time and the fundamental mode is already near its critical state. With decreasing  $\Delta\mathcal{E}$ , the growth rates become even smaller and the values closest to zero have been obtained for models 5, 6 and 7.

For model 7, the best parameterization was obtained and  $\Delta\mathcal{E} = 1.1$  was achieved. This still is clearly larger than for the simulation of rotating magnetoconvection for which  $\Delta\mathcal{E} = 0.28$  was found by Schinnerer et al. (2007). Figure 3 displays  $\mathcal{E}$  in comparison to  $\mathcal{E}^{(1)}$ . Differences are visible in all components, but also principal similarities.

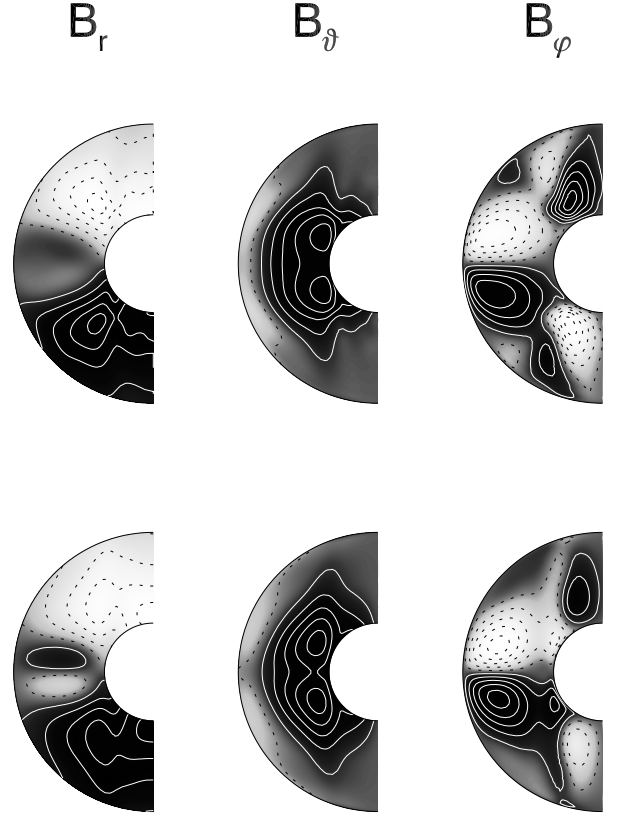
Figure 4 compares the azimuthally and time-averaged magnetic field resulting from direct numerical simulations with the fundamental eigenmode of  $\mathbf{D}$  for model 7. Apart from small differences, direct numerical simulations and mean-field calculations agree very well for this example.



**Fig. 3.** Contour plots of  $\mathcal{E}_r$ ,  $\mathcal{E}_\theta$ , and  $\mathcal{E}_\phi$  (top line, from left to right) and  $\mathcal{E}_r^{(1)}$ ,  $\mathcal{E}_\theta^{(1)}$ , and  $\mathcal{E}_\phi^{(1)}$  (bottom line, from left to right) for model 7 in units of  $(\nu/L)(\rho\mu\eta\Omega)^{1/2}$ . In these units, contour lines correspond to  $\pm 3, \pm 9, \pm 15$ .

## 6. Discussion

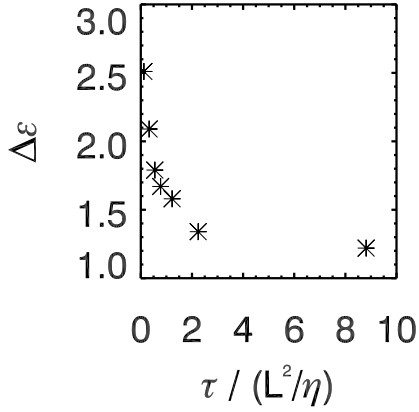
We do not derive mean-field coefficients for time-averaged mean fields in a formal sense. Instead, the comparisons in Fig. 3 and Fig. 4 are motivated by assumptions i) and ii) in Section 3. They are reasonably well fulfilled for the simulations considered here. For model 7,  $\nu$  time-averaged over 30 magnetic diffusion times drops to 0.1% of  $V$ . A similar decrease has been found for  $b$ . Moreover, the axisymmetric flow and the axisymmetric magnetic field show comparatively little time variation. The time-dependent components of  $\bar{V}$  and  $\bar{B}$  are of the order of 8% (37%) and 17% (23%) of the total (axisymmetric) flow and the total (axisymmetric) magnetic field, respectively. In other words, time-averaging results in axisymmetric fields and azimuthal averaging reduces somewhat the time variability. Combining both leads to an extended averaging and thus to an improved scale separation. This finding is also supported by Fig. 5 and Fig. 6. Figure 5 shows a noticeable decrease of  $\Delta\mathcal{E}$  for model 5, if the time-averaging period  $\tau$  is increased.  $\Delta\mathcal{E}$  drops rapidly until  $\tau \approx 2L^2/\eta$ ; an extension of the time-averaging interval over more than 2 magnetic diffusion times improves the scale separation only slightly. Moreover, figure 6 compares the residual with the mean magnetic field for model 1 (upper panel) and model 7 (lower panel). In contrast to model 1, the time-dependent residual and the time-averaged mean magnetic field of model 7 vary on fairly different length scales. Thus, a bet-



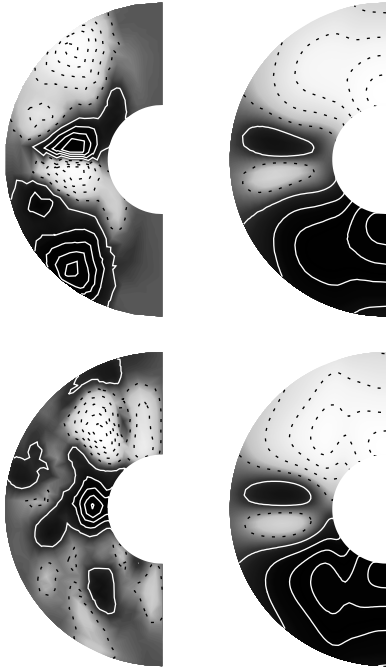
**Fig. 4.** Comparison between direct numerical simulations and mean-field calculations for model 7. Bottom line: azimuthally and time-averaged magnetic field obtained by solving (9-11). Top line: fundamental eigenmode resulting from (15). Each component has been normalised with respect to its absolute maximum. Therefore, the greyscale coding varies from  $-1$ , white, to  $+1$ , black, and the contour lines correspond to  $\pm 0.1, \pm 0.3, \pm 0.5, \pm 0.7, \pm 0.9$ .

ter scale separation for the time-dependent models seems to be plausible. Consequently,  $\Delta\mathcal{E}$  drops drastically from model 1 to model 2. However, there are still noticeable differences between  $\mathcal{E}$  and  $\mathcal{E}^{(1)}$  in Fig. 3 for model 7. From this comparison alone, it would be difficult to decide, whether the parameterization of  $\mathcal{E}$  is already satisfactory. Further support for its reliability comes from the good agreement between direct numerical simulations and mean-field modeling in Fig. 4. Contour plots of all three components of the leading eigenmode of  $D$  agree well with corresponding results from direct numerical simulations. Moreover, the growth rates of the leading eigenmodes are close to zero, as it has to be expected for stochastically stationary dynamos in the kinematically stable regime.

Dynamo action in stars and planetary cores most probably takes place on a wide range of length and time scales. Due to computational limitations, numerical simulations do not cover the whole range of scales possibly involved. Global dynamo simulations focus on large scales only, and small-scale dynamo action as reported for box simulations (e.g. Vögler & Schüssler 2007) is typically not present. Therefore, the component of the electromotive force independent of the mean magnetic field, i.e.  $\mathcal{E}^{(0)}$  in (4), is zero for most global dynamo simulations. The



**Fig. 5.**  $\Delta\mathcal{E}$  for model 5 varying with increasing time-averaging period  $\tau$ .



**Fig. 6.** Upper panel: non-axisymmetric radial magnetic field at arbitrary longitude (left) and azimuthally-averaged radial magnetic field (right) for model 1. Lower panel: non-axisymmetric and time-dependent radial magnetic field at arbitrary longitude and time (left) and azimuthally and time-averaged radial magnetic field (right) for model 7. The style of the contour plots is the same as in Fig. 4

possibility of a contribution to  $\mathcal{E}$  from small-scale dynamo action has been intensively discussed in the literature (e.g. Rädler 1976; Rädler 2000; Rädler & Rheinhardt 2007). Recently, it has been used to argue against the applicability of mean-field theory (Cattaneo & Hughes 2009). However, conceptual difficulties which might result from simultaneous small- and large-scale dynamo action are at present not relevant for the approach followed here.

The problem of scale separation has been addressed in this study with regard to spatial scales, only. This seems to be appropriate because we intend to describe stochastically stationary mean-fields with zero growth rate resulting from self-consistent

dynamo simulations. More generally, time derivatives of the mean field in the expansion of  $\mathcal{E}$  have to be taken into account. Similarly to non-local effects (Brandenburg et al. 2008), also memory effects of the flow may influence the parameterization of the electromotive force (Hubbard & Brandenburg 2009; Hughes & Proctor 2010).

Mean-field theory as presented here is intrinsically a kinematic approach. In general, an eigenvalue calculation based on (15) will lead to growing modes, even if the mean-field coefficients are derived from a saturated velocity field (Tilgner & Brandenburg 2008; Cattaneo & Tobias 2009). In such a situation, mean-field models might not match direct numerical simulations, unless a more self-consistent extension of the theoretical framework and the test-field method is applied (Courvoisier et al. 2010; Rheinhardt & Brandenburg 2010). However, the class of chaotically time-dependent dynamos we considered is kinematically stable. Schinnerer et al. (2010a) identified a regime of fast rotating dynamos which are dominated by only one dipolar mode at marginal stability, whereas all overtones are highly diffusive. A saturated velocity field from this class of dynamos does not lead to exponential growth of the magnetic field in a corresponding kinematic calculation. Consequently, the mean-field approach based on (15) is applicable for this dynamo regime, as confirmed once more in this study. In particular for models beyond this regime, the reliability of the mean-field approach presented here is not guaranteed and has to be made plausible by a comparison with direct numerical simulations (e.g. Schinnerer et al. 2011a).

## 7. Summary

The mean-field description of a (quasi-)stationary dynamo suffered from a poor scale-separation and therefore from an insufficient parameterization of the electromotive force (Schinnerer et al. 2007). For the chaotically time-dependent dynamos considered here, both improves a lot, if a combined azimuthal and time average is applied. The more accurate parameterization of  $\mathcal{E}$  leads to a good agreement between mean-field models and direct numerical simulations: Field topologies and growth rates resulting from both approaches are very similar.

In conclusion, the test-field method to determine mean-field coefficients from direct numerical simulations proves to be reliable for chaotically time-dependent dynamos, too. Mean-field theory may serve as a powerful tool to analyse dynamo processes in global models resulting from direct numerical simulations.

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